Randomness and Mr. Goodman's Paradox

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A necessary condition for the justification of any inductive argument of the form

(i) For all x observed before time T_n, if x has been observed to be E, then x has been observed to be G.

(ii) φ will be observed at T_{n+k} to be E.

It is probable therefore that

(iii) φ will be observed at T_{n+k} to be G.

is that the sampled objects in (i) and the reference object φ in (ii) and (iii), prior to being observed to be G or not-G, must not be known to possess any property that is known to be a necessary condition for an object possessing property G and not property not-G (or alternatively possessing property not-G and not property G).

If, for example, one should wish to determine whether or not an arbitrarily chosen Lilliputian is likely to be monogamous, and one knows that monogamous Lilliputians, unlike their polygamous compatriots, are required by law to live in two-story houses, one must not restrict one's sampled and reference objects to those Lilliputians living in two-story houses, or one's selection will be biased and one's induction unjustified. By satisfying the randomness condition, one would not insure the randomness of the selection, for the condition is necessary but not sufficient. By not satisfying the condition, however, one would insure the bias of the selection and hence the unjustifiability of the consequent induction.

(Randomness thus stands to bias in a relationship similar to that in which verification stands to falsification, and their logics evidence a similar Popperian asymmetry. Though and can never verify a universal proposition, one may be able to falsify it. Similarly, though one can never insure the randomness of a selection of sampled or referenced objects, one may be able to demonstrate its bias.)

Consider now any instantiation of the above argument upon which Goodman's paradox rests, for example:

(1) For all x observed before midnight 31 December 2074, if x has been observed to be an emerald, then x has been observed to be grue.

(2) φ will be observed at noon on 01 January 2075 to be an emerald.

It is probable therefore that

(3) φ will be observed at noon on 1 January 2075 to be grue.[1]

Upon reflection, it is apparent that the argument cannot be inductively justifiable, for given the definition of 'grue', the randomness condition on the selection of the sampled objects in (1) and the reference object φ in (2) and (3) cannot be satisfied.

$$x \text{ is grue} \overset{\text{def}}{=} x \text{ exists before the advent of the year 2050 and is green, or } x \text{ exists after the advent of the year 2050 and is blue}^{2}$$

By premise (1) and the definition of 'grue', all of the sampled emeralds have been observed to be green, and, having been observed to be green *before* the advent of the year 2075, have been observed to be grue. Had any of the sampled emeralds been observed to be green but been observed to be green *after* the advent of the year 2075, they could not – by definition – have been observed to be grue. Being observed to be green, in other words, is insufficient for determining the grueness of the sampled emeralds; they must have been observed to be green *before the advent of the year 2075*. Having the property

existing before the advent of the year 2075

is thus a necessary condition for the sampled green emeralds being grue and not being not-grue. But since the sampled emeralds were selected from only those emeralds known to exist *before* the advent of the year 2075, each of the sampled emeralds must be known to possess a property that is a necessary condition for it being grue and not being not-grue, prior to its grueness being observed, thus violating the randomness condition.

Similarly, by premise (2) and the definition of 'grue', the reference object φ will be observed to be an emerald *after* the advent of the year 2075. If the conclusion (3) of the

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[1] See Nelson Goodman, *Fact, Fiction, and Forecast* (2nd edition) (Indianapolis, Indiana: Bobbs-Merrill Company, 1965), pages 72-81.

[2] Ibid., page 74. In Goodman's words, the predicate 'grue' "applies to all things examined before t just in case they are green but to other things just in case they are blue".

argument should be true, however, then φ will also be observed to be blue *after* the advent of the year 2075 and hence be observed to be grue.[3] Were φ to be observed to be blue *before* the advent of the year 2075, it could not – by definition – be observed to be grue. Being observed to be blue, in other words, is insufficient for determining the grueness of φ; φ must be observed to be blue *after the advent of the year 2075*. Having the property

existing after the advent of the year 2075

is thus a necessary condition for the blue φ being grue and not being not-grue. But since φ will be selected from only those objects known to exist *after* the advent of the year 2075, φ must be known to possess a property that is a necessary condition for it being grue and not being not-grue prior to its grueness being observed, thus violating the randomness condition.

Every sampled object in (1) and the reference object φ in (2) and (3), therefore, must be known to possess at least one property that is known to be a necessary (though not sufficient) condition for it being grue and not being not-grue, prior to its grueness being assessed, thus violating the randomness condition.

Although it is true, therefore, that the "temporal quality" of the predicate 'grue' is "merely accidental" (in the sense that there are many properties other than temporality about which systematic violations of the randomness condition could be constructed for any predicate, including 'grue'), the truth is irrelevant to the flaw in the induction; for Goodman has violated the randomness condition that 'grue' imposes, accidentally or otherwise, through the temporal character *explicit in its definition*.[4] Were it to be defined differently, but were its definition to impose a similar bias upon the sampled or reference objects denoted in the premises of an induction, the resulting inference would be equally (and equally obviously) unjustifiable.

Nor is Goodman's vaunted "relativity" argument sufficient to sustain the purported paradox.[5] Since all inductions make predictions about reference objects that must be confirmed or disconfirmed through observations *made later* than the observations on the sampled objects (predictions, that is, that must be confirmed or disconfirmed in the future), it should be apparent from the above discussion that all predicates having a

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[3] If the conclusion of the argument should be false, the resulting induction could be shown to be unjustifiable by a formally similar argument.

[4] The quoted phrases are from Jay Hullett and Robert Schwartz, "Grue: Some Remarks", *The Journal of Philosophy*, Volume 64, 11 May 1967, page 261.

[5] The word 're1ativity" is Goodman's own. See Goodman, op. cit., pages 79-81.

defined temporal quality like 'grue' must be inductively useless, for they cannot be used in inductions without violating the randomness condition.[6] It is impossible, in other words, to select a random sample when using such predicates inductively. (Only God, existing outside time itself, could select a temporally random sample of objects *in time* having the property, for example, of being 'grue'. But God, existing outside time itself and hence able to select such a random sample, would have no need to make inductions.) To point out, as Goodman does, that the predicates 'green' and 'blue' would be similarly defective to users of the 'grue'-'bleen' language, is both true and irrelevant.[7] Users of such a language would indeed be prohibited from using 'green' and 'blue' in inductions on pain of violating the randomness condition, but they would also be prohibited from using 'grue' and 'bleen' for the same reason! They would therefore be prohibited from making justifiable predictions about *either* the greenness, blueness, grueness or bleenness of emeralds, whereas users of the 'green'-'blue' language, prohibited only from using 'grue' and 'bleen', are free to construct justifiable inductions about the predicted greenness or blueness of emeralds to their hearts' content.

If one systematically violates an obvious and necessary condition for the justifiability of an induction, as Goodman has done, one can hardly be ingenuously surprised when the resulting inference contradicts a non-violating induction (the induction that results, for example, when the predicate 'green' is substituted for the predicate 'grue' in premises (1) and (3) above). From a suitably biased sample, a suitably contradictory inference *of course* can be drawn. But the obvious bias of the sample precludes paradox however contradictory the resulting inference. The puzzle concerning 'grue', therefore, is not that its use results in contradictory inferences, but rather that so many philosophers have been surprised by the fact.

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[6] The predictions that are confirmed or disconfirmed by future evidence may, of course, convey information about reference objects existing only in the past. The confirmations and disconfirmations of the predictions, however, must come later than the observations on the sampled objects for an induction to have occurred.

[7] 'bleen', in Goodman's words, "applies to emeralds examined before time t just in case they are blue and to other emeralds just in case they are green". See Goodman, op. cit., page 79.